# Connectivity in wireless sensor networks*

Patrick Thiran (EPFL, Lausanne)
(joint work with Olivier Dousse and François Baccelli)

As is it an essential feature, connectivity has received quite a lot of attention in the previous decade already, in the context of packet radio networks, and has gained renewed interest recently in the context of ad-hoc and sensor networks. Most results apply to the full connectivity of a network made of a finite number of nodes.

Allowing some nodes to remain isolated enables to reduce significantly the maximal range that nodes need to have. In a sensor network, the full connectivity of the network may be less essential than the power savings that can be obtained by reducing the radio range, provided that the area on which the network is deployed is well covered (coverage problem) and that the base station collecting all information is connected to most sensors, no matter how distant they are from the base station (connectivity problem).

In this presentation, we will address the connectivity problem. We assume thus that the number $N$ of sensors is not fixed nor on a finite area, but that they are given as points of a Poisson process over the plane. We do not make assumptions on its intensity $\lambda$, so that the studies also apply to areas covered by a low density of sensors, or to areas where an important number of sensors are turned off. The problem is then related to percolation theory. The percolation probability is the probability that an arbitrary sensor (in particular, the base station collecting all data) belongs to a cluster of infinite size over the entire plane. The main results of percolation theory is that there exists a finite, positive value $\lambda_c$ of $\lambda$, under which the percolation probability is zero (sub-critical phase) and above which it is non zero (super-critical phase). In the sub-critical phase, all clusters are almost surely finite, and their size has is a random variable whose distribution has an exponentially decreasing tail. In the super-critical phase, the percolation probability usually increases quite rapidly with $\lambda$.

Up to now, we have used a very simple model, the Boolean model, where two nodes are connected to each other if and only if their distance is less than some value. In reality, interferences need to be taken into account. We consider therefore a model (the physical model) where two nodes are connected to each other if and only if the signal to noise ratio at the receiver is larger than some prescribed value. The noise is the sum of the contribution of interferences from all other nodes, weighted by a coefficient $\gamma$, and of a background noise.

*Extended Abstract for "NeXtworking'03" First COST-IST(EU) - NSF(USA) Workshop on Exchanges & Trends in Networking, June 23-25, 2003, Chania, Crete, Greece

The problem is more complex, as now direct connections between two particular nodes depend on other nodes positions, contrary to the Boolean model. Moreover, two parameters ($\gamma$, $\lambda$) instead of one ($\lambda$) are now to be considered.

We find that there is a critical value of $\gamma$ above which the network is made of disconnected clusters of nodes. We also prove that if $\gamma$ is non zero but small enough, there is a node spatial density (the percolation threshold) above which the network contains a large (theoretically infinite) cluster of nodes, enabling distant nodes to communicate in multiple hops. We also comment on the existence of a super-critical phase at large densities $\lambda$. Finally, we discuss the impact of these findings on simple MAC schemes.